(7) If resistor R

$$P = \frac{E^2 R}{(R+r)^2}$$

If E and r are fixed but R varies what is the maximum value of the power

$$P = \frac{E^2 R}{(R+r)^2}$$

$P = E^2 R\,(R+r)^{-2}$ but r is fixed

$$P'(R) = E^2\left(-2(R+r)^{-3} \cdot \cancel{\times}\right)$$

$$P'(R) = \frac{-2E^2 \times 1}{(R+r)^3}.$$

$$\frac{-2E^2}{(R+r)(R+r)(R+r)}$$

$$(R+r)(R+r) = (R^2+2Rr+r^2)(R+r)$$

$$= R^3 + 2R^2 r + Rr^2 + R^2 r + 2Rr^2 + r^3$$

$$= R^3 + 3R^2 r + 3Rr^2 + r^3.$$

maximum power $= \dfrac{-2E^2}{R^3+3R^2 r+3Rr^2+r^3}$

(8) Evaluate the definite integral

(a) $\int_0^8 \left(\frac{1}{8} + \frac{1}{2}w\right) dw$

$= \int_0^8 \frac{1}{8}\, dw + \int_0^8 \frac{1}{2} w\, dw$

$= \left[\frac{1}{8} w\right]_0^8 + \left[\frac{1}{4} w^2\right]_0^8$

$= [1 - 0] + [16 - 0)$

$= 17.$

(b) $\int_{\frac{\pi}{6}}^{\pi/2} \csc t \cot t\, dt$

$= \csc(t) \cot(t) = \frac{1}{\sin(t)} \cdot \frac{\cos t}{\sin t}$

$= \int_{\frac{\pi}{6}}^{\pi/2} \frac{\cos t}{\sin^2 t}\, dt = \int_{\frac{\pi}{6}}^{\pi/2} \cos t \cdot \int_{\pi/6}^{\pi/2} \frac{1}{\sin^2 t}.$

$= -\csc(t) + c$

lim y csc(t)

$\lim_{t \to \frac{\pi}{6}} -\csc(t) = -2$

$\lim_{t \to \pi/2} -\csc(t) = 1$

F(b) − f(a)

$= -1 - (-2) = \underline{\underline{1}}$

(e) $$\int_0^1 (5x - 5^x)\,dx$$

$$= \int_0^1 5x\,dx - \int_0^1 5^x\,dx$$

$$= \left[\frac{5x^2}{2}\right]_0^1 - \left[\cancel{5\frac{x^2}{2}}\ 5^{x/2}\right]_0^1 \cancel{dx}.$$

$$= 5/2 - \left[5^{1/2} - 1\right]$$

$$5/2 - (\sqrt{5} - 1]$$

$$5/2 - (2.24 - 1)$$

$$5/2 - 1.24$$

$$= \underline{\underline{1.26}}$$

(9) Evaluate indefinite integral

(a) $\int x^2\sqrt{x^3+1}\,dx$

let $u = x^3+1$

$\frac{du}{dx} = 3x^2$, $du = 3x^2 dx$

$x^2 dx = \frac{du}{3}$

$\int x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int u^{1/2} du.$

$= \frac{1}{3} \frac{u^{3/2}}{3/2}$

$= \frac{2}{9} u^{3/2}$

$= \frac{2}{9}(x^3+1)^{3/2} + C$

(b) $\int \sin x \sin(\cos x)\,dx$

let $u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$= -\int \sin(u)\,du$

$= \int -\sin(u)\,du$

$= \cos(u) + C$ but $u = \cos x$

$= \cos(\cos x) + C$

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